

# Synthesis of Intelligent Control of Traffic Flows in Urban Roads Based on the Logical Network Operator Method

A.I. Diveev, E.A. Sofronova

**Abstract**— A problem of optimal control of traffic flows in the urban network is considered. A derivation of the mathematical model of control by the traffic lights at intersections is given. The mathematical model of control object is obtained using the controlled networks theory. This model is a system of nonlinear finite-difference equations. To present a large scale road networks the model contains the connection matrices. The connection matrices describe connections between input and output roads in subnetworks. These matrices allow considering the influence of traffic flows in one subnetwork the ones in other subnetwork.

The traffic flow control is done by the coordination of active phases of traffic lights. The optimal control problem is to minimize the difference between the total input flow and total output flow for all subnetworks. To solve the optimal control synthesis problem we use a logical network operator method. An example of model for the network of three subnetworks is given.

## I. INTRODUCTION

THE influence of active phases of traffic lights at neighboring intersections is significant when the road network is busy. If the active phases are not properly coordinated then the traffic jams occur, and as a result the intersections can no longer be controlled. Thus the effective traffic flow control problem can be solved by the active phases coordination.

The problem of active phase coordination for the traffic lights at several intersections on different roads is still unsolved. One of the reasons for it is the absence of adequate mathematical models of traffic flow control by the traffic lights. Nowadays it is considered that the hydraulic models are the most appropriate ones [1-5]. These models are the systems of differential equations in partial derivatives. We should also note that these models do not explicitly include the traffic light control and are quite complex for optimization.

Other models [6] do not describe direct dependence of flow parameters from the duration of active phases. For example, in model [6] the density of traffic flow is

controlled. The model describes large scale networks and determines the change of flow parameters by nonlinear system in continuous and discrete time. This system does not describe the road configuration that depends on the active phases at intersections.

In this paper a derivation of traffic flow control mathematical model on the basis of the controlled networks theory is given [7-9]. The controlled networks are presented as graphs of flexible configurations. The obtained mathematical model is a system of nonlinear finite-difference equations.

In works [7-9] to derive a model it is necessary to use a graph of road network. When considering a large scale network we obtain a large scale graph that leads to computational complexity. In this work we propose an improved approach. In the new model a large scale network is presented as several connected subnetworks. Each subnetwork is described by the graph of smaller dimension.

The optimal control problem is stated for this system. To solve the problem of control synthesis we control the durations of active phases of traffic lights. As a result we obtain the dependence of control from the traffic flow state. An optimization criterion is the difference between the flows on the input and output roads.

## II. MATHEMATICAL MODEL OF TRAFFIC FLOWS

To construct a mathematical model of traffic flow control we use a directed graph of flexible structure. The nodes of the graph are the parts of the road between intersections. The edges of the graph are maneuvers between parts of the road. Thus we obtain a directed graph of the road network.

Let the network consists of  $M$  intersections and  $L$  parts of the road. Maneuvers are performed at intersections. The active phases of traffic lights at intersections prohibit certain maneuvers and each state of traffic lights determines a part of the graph of the road network. As a result we obtain a graph of flexible configuration. To present the graph we use the following matrices:

- an adjacency matrix of basis network graph

$$\mathbf{A} = [a_{ij}], \quad a_{ij} \in \{0, 1\}, \quad i, j = \overline{1, L}; \quad (1)$$

- a capacity matrix

$$\mathbf{B} = [b_{ij}], \quad b_{ij} \in \mathbf{R}_+^1 \cup \{0\}, \quad (2)$$

where  $b_{ij}$  estimates the flow from road  $i$  to road  $j$  for some time interval;

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A.I. Diveev is with the Institution of Russian Academy of Sciences Dorodnicyn Computing Centre of RAS (phone: +7(905)711-44-27; e-mail: aidiveev@mail.ru).

E.A. Sofronova is with Cybernetics and Mechatronics Department at Peoples' Friendship University of Russia (e-mail: sofronova\_ea@mail.ru).

- a control matrix

$$\mathbf{C} = [c_{ij}] , c_{ij} \in \{1, \dots, M\}, \quad (3)$$

where  $c_{ij}$  is a number of the intersection at which maneuver from road  $i$  to road  $j$  is performed;

- a distribution matrix

$$\mathbf{D} = [d_{ij}] , d_{ij} \in [0, 1] , \quad (4)$$

where  $d_{ij}$  indicates the part of the traffic flow at road  $i$  and performs maneuver to road  $j$ , for all parts of the road (5) should be satisfied

$$\sum_{j=1}^L d_{ij} = 1, i = \overline{1, L}; \quad (5)$$

- an allowable phase matrix

$$\mathbf{F} = [F_{ij}] , i, j = \overline{1, L}, F_{ij} = \{f_{ij1}, \dots, f_{ijk(c_{ij})}\} , \quad (6)$$

$$f_{ijk} \in \{0, u_{c_{ij}}^+\} , 1 \leq k(c_{ij}) ,$$

where  $u_{c_{ij}}^+$  is a maximal number of active phase at intersection  $c_{ij}$ ,  $k(c_{ij})$  is a maximal quantity of traffic light phases that permits maneuver from road  $i$  to road  $j$  at intersection  $c_{ij}$ ,  $F_{ij}$  is a set of numbers of the phases that permit maneuver from road  $i$  to road  $j$ . All matrices have identical structure:  $b_{ij} > 0$ ,  $c_{ij} > 0$ ,  $d_{ij} > 0$ ,  $F_{ij} \neq \emptyset$ , if  $a_{ij} = 1$ , otherwise  $b_{ij} = 0$ ,  $c_{ij} = 0$ ,  $d_{ij} = 0$ ,  $F_{ij} = \emptyset$ .

To describe the flexibility of the network configuration we introduce the control vector

$$\mathbf{u} = [u_1 \dots u_M]^T, u_i \in \{0, \dots, u_i^+\} , \quad (7)$$

where  $u_i$  is a number of the phase of traffic light at intersection  $i$ ,  $u_i^+$  is a maximal number of active phase at intersection  $i$ ,  $i = \overline{1, M}$ .

The change of network configuration is described by the configuration matrix that is also an adjacency matrix of a partial subgraph

$$\mathbf{A}(\mathbf{u}) = [a_{ij}(u)] , \quad (8)$$

$$a_{ij}(u) = \begin{cases} 1, & \text{if } a_{ij} = 1, u_{c_{ij}} \in \{F_{ij}\} \\ 0, & \text{otherwise} \end{cases}$$

A configuration matrix influences the structures of all other matrices.

To describe the parameters of the traffic flow let us introduce a time interval  $\Delta t$ . We assume that the durations of all phases are estimated in integer number  $\Delta t$ . We also assume that all traffic lights are synchronized so that the count of integer number of time intervals for all traffic lights in the network is done simultaneously. To obtain the quantitative characteristics of the traffic flow for each part of

the road we use a flow vector

$$\mathbf{x}(t_k) = [x_1(t_k) \dots x_L(t_k)]^T , \quad (9)$$

where  $x_j(t_k)$  is a number of cars on the road  $j$  at time  $t_k$ ,  $x_j(t_k) \in \mathbf{R}^1$ ,  $j = \overline{1, L}$ ,  $k = \overline{0, N}$ ,  $N$  is a number of control timing periods.

Further on rewrite

$$x_j(t_k) = x_j(k) . \quad (10)$$

Traffic flow  $\mathbf{x}(k)$  depends on the road network configuration and the values of flow vector at the previous moment of time  $\mathbf{x}(k-1)$ .

Suppose that all cars perform maneuvers at one timing period simultaneously. Maneuvers are performed in two steps. At the first step the cars leave the part of the road where they have been to perform a maneuver. At the second step they finish the maneuver and go to other roads.

At the first step the number of cars is decreased by the number of cars that performed the maneuver as

$$\mathbf{x}\left(k - \frac{1}{2}\right) = \mathbf{x}(k-1) - \min\left\{\Delta\mathbf{x}'\left(k - \frac{1}{2}\right), \Delta\mathbf{x}''\left(k - \frac{1}{2}\right)\right\} , \quad (11)$$

where  $\Delta\mathbf{x}'\left(k - \frac{1}{2}\right)$  is a number of cars that needs to perform

a maneuver,  $\Delta\mathbf{x}''\left(k - \frac{1}{2}\right)$  is a number of cars that can

perform the maneuvers for one timing period according to the capacity of the road,

$$\Delta\mathbf{x}'(k-1/2) = ((\mathbf{x}(k-1)\mathbf{1}_L^T) \odot \mathbf{D} \odot \mathbf{A}(\mathbf{u}(k))\mathbf{1}_L) , \quad (12)$$

$$\Delta\mathbf{x}''(k-1/2) = (\mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B})\mathbf{1}_L , \quad (13)$$

$\mathbf{1}_L^T = [1 \dots 1]$ ,  $\odot$  is an Hadamard product of matrices,

$$\mathbf{x}(k-1/2) = [x_1(k-1/2) \dots x_L(k-1/2)]^T ,$$

$$\Delta\mathbf{x}''(k) = [\Delta x_1'' \dots \Delta x_L'']^T , \Delta\mathbf{x}'(k) = [\Delta x_1' \dots \Delta x_L']^T .$$

Present (11) as

$$\mathbf{x}(k-1/2) = \mathbf{x}(k-1) - (\Delta\mathbf{x}'(k-1/2) - (\Delta\mathbf{x}'(k-1/2) \div \Delta\mathbf{x}''(k-1/2))) ,$$

where

$$a \div b = \begin{cases} a-b, & \text{if } a > b, \\ 0, & \text{otherwise.} \end{cases}$$

At the second step the change of the traffic flow is described as

$$\mathbf{x}(k) = \mathbf{x}(k-1/2) + \min\{\Delta\mathbf{x}'(k), \Delta\mathbf{x}''(k)\} , \quad (14)$$

or

$$\mathbf{x}(k) = \mathbf{x}(k-1/2) + \Delta\mathbf{x}'(k) - (\Delta\mathbf{x}'(k) \div \Delta\mathbf{x}''(k)) , \quad (15)$$

where

$$\Delta\mathbf{x}'(k) = ((\mathbf{x}(k-1)\mathbf{1}_L^T) \odot \mathbf{D} \odot \mathbf{A}(\mathbf{u}(k)))^T \mathbf{1}_L , \quad (16)$$

$$\Delta\mathbf{x}''(k) = (\mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B})^T \mathbf{1}_L . \quad (17)$$

As a result we obtain the following model of traffic flow control

$$\begin{aligned} \mathbf{x}(k) = & \mathbf{x}(k-1) - \left( (\mathbf{x}(k-1) \mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} - \right. \\ & - \left( (\mathbf{x}(k-1) \mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} \div \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B} \right) \mathbf{1}_L + \\ & + \mathbf{B} \mathbf{1}_L + \left( (\mathbf{x}(k-1) \mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} - \right. \\ & - \left. \left( (\mathbf{x}(k-1) \mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} \div \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B} \right) \right)^T \mathbf{1}_L + \boldsymbol{\delta}(k), \end{aligned} \quad (18)$$

where  $\boldsymbol{\delta}(k) = [\delta_1(k) \dots \delta_L(k)]^T$ ,  $\delta_i(k)$  is the value of input flow at road  $i$ ,  $i = \overline{1, L}$ , depending from some random factor.

Consider a large road network of  $K$  subnetworks. The models of subnetworks are presented as:  $(\mathbf{A}^l(\mathbf{u}(k)), \mathbf{B}^l, \mathbf{C}^l, \mathbf{D}^l, \mathbf{F}^l : l = \overline{1, K})$ .

To connect the models of all subnetworks let us introduce a connection matrix for each subnetwork

$$\mathbf{R}^l = [r_{i,j}^l], \quad i = \overline{1, n_1^l}, \quad j = \overline{1, K}, \quad l = \overline{1, K}, \quad (19)$$

where  $r_{i,j}^l$  is an index of element in the input roads vector for the road  $j$ ,  $n_1^l$  is a number of output roads in the subnetwork  $l$ .

For each part of the road the model should include vectors of input and output roads.

$$\mathbf{v}^l = \begin{bmatrix} v_1^l & \dots & v_{n_0^l}^l \end{bmatrix}^T, \quad (20)$$

$$\mathbf{w}^l = \begin{bmatrix} w_1^l & \dots & w_{n_1^l}^l \end{bmatrix}^T, \quad (21)$$

where  $v_i^l$  is an index of an input road in subnetwork  $l$ ,  $w_i^l$  is an index of an output road in subnetwork  $l$ ,  $n_0^l$  is a number of input roads in subnetwork  $l$ .

Using connection matrices we can simulate the flow dynamics in all subnetworks simultaneously. At each time interval  $\Delta t$  we recalculate the flow vector in accordance with connection matrix

$$\forall r_{i,j}^l, j = \gamma \neq 0, \quad x_{\alpha}^j(k) = x_{\beta}^l(k), \quad (22)$$

where  $i = \overline{1, n_1^l}$ ,  $j = \overline{1, K}$ ,  $l = \overline{1, K}$ ,  $\alpha = v_{\gamma}^j$ ,  $\beta = w_i^l$ .

### III. THE PROBLEM OF OPTIMAL CONTROL OF TRAFFIC FLOWS

The traffic flow control is performed by the change of active phase durations at controlled intersections in the considered network. It should be taken into consideration that the phases are switched in the certain order, and the duration of each phase is limited. A phase duration can be estimated in the number of time intervals  $\Delta t$ . Let us introduce a vector of sets of minimal phase durations

$$\mathbf{Q} = \begin{bmatrix} \begin{pmatrix} q_{1,1}, \dots, q_{1,u_1^+} \end{pmatrix} \\ \dots \\ \begin{pmatrix} q_{M,1}, \dots, q_{M,u_M^+} \end{pmatrix} \end{bmatrix}, \quad (23)$$

where  $q_{j,i}$  is a duration of active phase  $i$  at intersection  $j$ ,  $i = \overline{1, u_j^+}$ ,  $j = \overline{1, M}$ .

Suppose that at the moment  $k$  the phase  $i$  is active at intersection  $j$ , then

$$u_j(k) = i. \quad (24)$$

A phase switch condition is formulated as

$$s_j(k) = \begin{cases} s_j(k-1) + 1, & \text{if } s_j(k-1) < q_{j,i} \\ 0, & \text{if } i = (i-1) \bmod u_j^+ + 1 \end{cases}, \quad (25)$$

where  $s_j(k)$  is a duration of active phase  $i = u_j(k)$  at the moment  $k$  at intersection  $j$ .

To solve the optimal control problem for the traffic flows it is necessary to calculate the active phases durations of the traffic lights at intersections

$$q_{j,i}^- \leq q_{j,i} \leq q_{j,i}^+, \quad i = \overline{1, u_j^+}, \quad j = \overline{1, M}, \quad (26)$$

where  $q_{j,i}^-$  and  $q_{j,i}^+$  are given.

The control should minimize the functional

$$J_1 = \sum_{i \in I_1} x_i(N) - \sum_{j \in I_2} x_j(N) \rightarrow \min, \quad (27)$$

where  $I_1$  is a set of input roads numbers,  $I_2$  is a set of output roads numbers.

All values of flows should be constrained by

$$x_i(k) \leq x_i^+, \quad i = \overline{1, L}, \quad k = \overline{1, N}. \quad (28)$$

Let us add the constraints to the functional

$$\begin{aligned} J_1 = & \sum_{i \in I_1} x_i(N) - \sum_{j \in I_2} x_j(N) + \\ & + p \sum_{k=1}^N \sum_{i=1}^L \left( \left( \frac{x_i(k)}{x_i^+} - 1 \right) + \left| \frac{x_i(k)}{x_i^+} - 1 \right| \right) x_i^+ \rightarrow \min, \end{aligned} \quad (29)$$

where  $p$  is a penalty coefficient.

In our case (29) allows coping with the input and output roads where the number of cars is not limited. If the number of cars on the road is limited then  $x_i^+ \geq 0$ . If the number of cars does not exceed limitation  $x_i(k) \leq x_i^+$ , then

$$\left( \frac{x_i(k)}{x_i^+} - 1 \right) + \left| \frac{x_i(k)}{x_i^+} - 1 \right| = 0. \quad (30)$$

If the limitation is exceeded then

$$\left( \frac{x_i(k)}{x_i^+} - 1 \right) + \left| \frac{x_i(k)}{x_i^+} - 1 \right| > 0. \quad (31)$$

For the roads without limitations we have  $x_i^+ = -1$ . In this case for any  $x_i(k)$  we get (30).

Usually input and output roads have no limitations. According to (29) we assume

$$x_i^+ = -1, \text{ if } i \in I_1 \cup I_2 \text{ or } x_i^+ = \infty. \quad (32)$$

#### IV. THE SYNTHESIS OF INTELLIGENT CONTROL

To solve the problem of control synthesis it is necessary to find the dependence between the active phase durations and the number of cars on the roads

$$q_{i,j} = g_{i,j}(\mathbf{x}), \quad j = \overline{1, u_i^+}, \quad i = \overline{1, M}, \quad (33)$$

where  $g_{i,j}(\mathbf{x})$  is a multidimensional function that describes dependence between the flow state on all roads and duration of phase  $j$  at intersection  $i$ .

To solve the control synthesis problem we use a network operator method [10, 11]. The method allows searching the structure and parameters of function by genetic algorithm. The structure of function is presented as a graph. The nodes and edges of the graph correspond to given sets of binary and unary operations.

In our case we are more interested in the value that shows how occupied the road is rather than in the absolute value of flow  $\mathbf{x}$  in it. We can estimate the occupancy of the road by discrete values from 0 to  $z^+$ . To estimate the traffic flows let us introduce an integer grid

$$Z = \{0, 1, \dots, z^+\}. \quad (34)$$

To estimate the flows on internal roads we take into consideration the limitations on these roads. The more the value  $\frac{x_i}{x_i^+}$ , the more is its estimation by the grid (34).

To avoid having too long phase durations let us introduce an integer grid for phases durations from 0 to  $y^+$

$$Y = \{0, 1, \dots, y^+\}. \quad (35)$$

To obtain a phase duration we use a given increment  $\Delta y_{i,j}$ .

The phase duration is found from

$$q_{j,i} = q_{j,i}^- + y_{j,i} \Delta q_{j,i}, \quad (36)$$

where  $y_{i,j} \in Y$ .

Thus to solve the control synthesis problem it is necessary to determine the discrete value of phase duration by the discrete value of flow. As a result we obtain a  $k$ -valued function

$$\mathbf{Y} = G(\mathbf{z}), \quad (37)$$

where  $\mathbf{Y} = [y_{i,j}]$ ,  $y_{i,j} \in Y$ ,  $i = \overline{1, M}$ ,  $j = \overline{1, u_i^+}$ ,  $\mathbf{z} = [z_1 \dots z_L]^T$ ,  $z_k \in Z$ ,  $k = \overline{1, L}$ .

To solve the problem of control system synthesis namely finding  $G(\mathbf{z})$  we use the logical network operator method.

#### V. THE LOGICAL NETWORK OPERATOR METHOD

Network operator method was developed for the synthesis of control systems [10, 11]. The network operator allows to describe mathematical expression in a form of directed graph. The edges of the graph relate to unary operations. The nodes of the graph relate to binary operations. The source nodes relate to the arguments of the mathematical expressions. The solution of problem is searched with the help of genetic algorithm.

The method of logical network operator was developed for the synthesis problem of intelligent control [9, 12].

The method of logical network operator uses unary and binary operations of multi-valued logic

$$O_1 = (\varphi_1(z), \dots, \varphi_8(z)), \quad (38)$$

$$O_2 = (\omega_0(z', z''), \dots, \omega_3(z', z'')), \quad (39)$$

where

$$\omega_0(z', z'') = \max\{z' \bmod z^+, z'' \bmod z^+\},$$

$$\omega_1(z', z'') = \min\{z' \bmod z^+, z'' \bmod z^+\},$$

$$\omega_2(z', z'') = (z' + z'') \bmod z^+,$$

$$\omega_3(z', z'') = (z' \cdot z'') \bmod z^+,$$

$$\varphi_1(z) = z \bmod z^+, \quad \varphi_2(z) = (z + 1) \bmod z^+,$$

$$\varphi_3(z) = \begin{cases} (z - 1) \bmod z^+, & \text{if } z > 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$\varphi_4(z) = z^+ - 1 - z \bmod z^+,$$

$$\varphi_5(z) = \begin{cases} 2z, & \text{if } 2z < z^+, \\ z^+ - 1 - (2z) \bmod z^+ & \text{otherwise,} \end{cases}$$

$$\varphi_6(z) = \begin{cases} 3z, & \text{if } 3z < z^+, \\ z^+ - 1 - (3z) \bmod z^+ & \text{otherwise,} \end{cases}$$

$$\varphi_7(z) = \left\lfloor \frac{z \bmod z^+}{2} \right\rfloor, \quad \varphi_8(z) = \left\lfloor \frac{z \bmod z^+}{3} \right\rfloor.$$

For  $z^+ = 2$  a set of operations of multiple-valued logic can be presented as operations of two-valued logic.

For a two-valued logic we have

$$\omega_0(z', z'') = \omega_2(z', z'') = z' \vee z'',$$

$$\omega_1(z', z'') = \omega_3(z', z'') = z' \wedge z'',$$

$$\varphi_1(z) = z,$$

$$\varphi_2(z) = \neg z.$$

In the memory of PC the network operator is presented as an integer matrix. The matrix has the same structure as an adjacency matrix of the network operator graph. The ones in nondiagonal elements of the adjacency matrix are replaced by the indices of unary operations, diagonal elements are

replaced by the indices of binary operations.

Consider a two-valued logic function

$$y = x_1 \wedge x_2 \vee \neg x_3 \vee \neg x_4.$$

The network operator for given example is presented in Fig. 1.

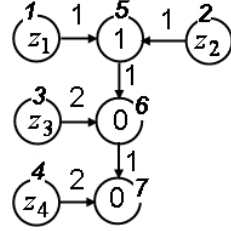


Fig. 1. The example of the logical network operator

The matrix of logical network operator has a form

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

To perform the synthesis the expert sets the initial working phase durations for different traffic flow values. For that purpose the expert simulates the traffic flow control using (18). The active phase durations should provide the optimal values of functional (27) or (29).

## VI. AN EXAMPLE

Consider a network of three subnetworks. The structure of connection is presented in Fig. 2. The network is divided taking into consideration control synthesis problem (37). We include in subnetwork all traffic flows that influence the phase durations in the subnetwork. Usually one subnetwork consists of neighboring intersections where phase durations influence each other specially when the traffic is intense.

The connection between subnetworks and the indexes of their input and output roads are presented in Fig. 2. For example, an output road 9 from the subnetwork 1 is connected to the input road 3 from the subnetwork 2, an output road 12 from the subnetwork 2 is connected to the input road 1 from the subnetwork 3.

For the given network the structures of subnetworks 1 and 3 coincide and are presented in Fig. 3. The structure of subnetwork 2 is depicted in Fig. 4.

Для подсетей имеем следующие входные и выходные векторы

$$\mathbf{v}^1 = \mathbf{v}^3 = [1 \ 2 \ 3 \ 4]^T, \quad \mathbf{w}^1 = \mathbf{w}^3 = [7 \ 8 \ 9 \ 10]^T, \\ \mathbf{v}^2 = [1 \ 2 \ 3 \ 4 \ 5]^T, \quad \mathbf{w}^2 = [8 \ 9 \ 10 \ 11 \ 12]^T.$$

Fig. 5 shows graph of subnetworks 1 and 3. Fig. 6 shows graph of subnetwork 2.

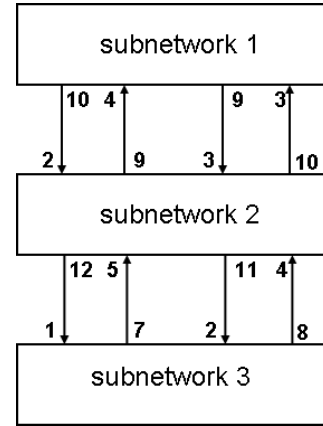


Fig. 2. A structure of subnetworks connection

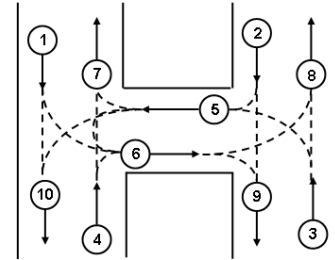


Fig. 3. A structure of subnetworks 1 and 3.

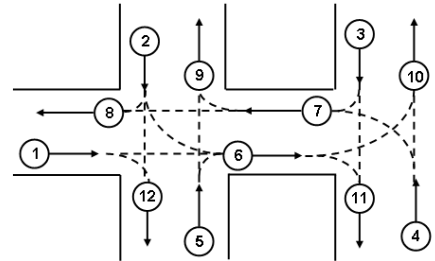


Fig. 4. A structure of subnetwork 2.

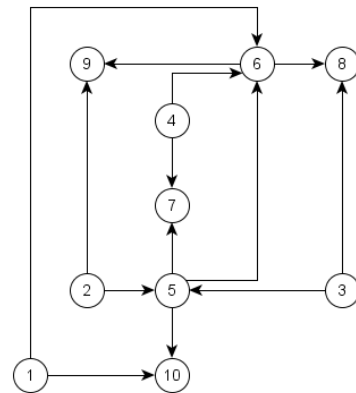


Fig. 5. A graph of subnetworks 1 and 3.

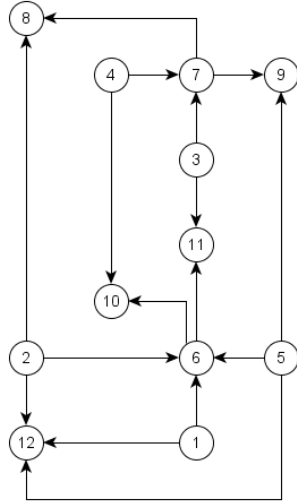


Fig. 6. A graph of subnetwork 2.

Adjacency matrices for graphs of the subnetworks are the following

$$A^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A^1 = A^3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The connection matrices for the given network are

$$R^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & 0 \end{bmatrix}, R^2 = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}, R^3 = \begin{bmatrix} 0 & 5 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

To construct a full model for each subnetwork I it is necessary to add a capacity matrix  $B^i$ , a control matrix  $C^i$ , a distribution matrix  $D^i$  and an allowable phase matrix  $F^i$  [8].

All matrices have the same structure as an adjacency matrix has. Indices of nonzero elements in these matrices correspond to nonzero elements of adjacency matrix.

According (8) each active phase of traffic lights prohibits certain maneuvers at intersections and changes the structure

of adjacency matrix. As a result we obtain a configuration matrix  $A(u)$  that determines the flow change in the network according to (18).

When we have solved the synthesis problem we get a  $k$ -valued logical function that determines durations of traffic lights phases in number of intervals.

The synthesis problem is solved for different timing periods during a day, for different week days and in case of change of network parameters.

The obtained model can be extended. To solve this problem we developed specific software for traffic flow simulation CTraf and NOP4C-S [13].

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